Bank Equity and Macroprudential Policy*

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Abstract

This paper investigates a new macroprudential policy in a DSGE model with financial frictions. As in Gertler, Kiyotaki and Queralto (2012), we propose to subsidize bank equities. The subsidy level is different, however. In the new policy, the subsidy is proportional to the bank capital ratio while in Gertler et al. (2012), it is a function of the shadow cost of the deposit. The new policy has two advantages: first, it is more applicable for practical policy design because the bank's balance sheet structure is an observable target for the central bank. Second, the new policy helps reduce both the moral hazard cost and the cost of bank borrowing from households. The results show that the new policy is welfare dominant.

Keywords: Macroprudential Policy, Bank Equity, Capital Ratio, DSGE Model

JEL classification: C61, E44, E58

1 Introduction

A low bank capital ratio on banks' balance sheets is an important factor for excessive systemic risk and financial instability. There is a great deal of literature that shows that individual banks tend to issue excessive short-term debts because of financial frictions and credit constraints[1], and banks undervalue the importance of the bank capital ratio for managing bank risks. As individual banks issue more short-term debts, they become more vulnerable to risks because short-term debts would be affected by credit crunch and the 'fire sales' effect. This feature of bank balance sheets has been highlighted by the recent financial crisis in 2009.

To improve financial stability, different macroprudential policy tools are designed. The literature on macroprudential policy instruments focuses on bank capital regulations

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[1]For example, Lorenzoni (2008), Bianchi (2009), Korinek (2011) and Stein (2011).

such as countercyclical capital requirement[2]. Gertler and Kiyotaki (2010) and Gertler, Kiyotaki and Queralto (2012) (GKQ hereafter) embed endogenous bank balance sheets in a DSGE model with financial frictions. In their framework, it is possible to compare the effectiveness of macroprudential policies which refers to the stability of the financial market and the reduction of real output loss.

In GKQ, a macroprudential policy is introduced to mitigate the real output loss caused by the financial shock. In their policy, the central bank taxes bank assets and offers subsidy to bank equities. The subsidy rate is designed to respond to the shadow cost of bank deposits. This policy induces bankers to choose a higher capital ratio in steady state, so it practically works as a capital ratio requirement and helps improve welfare. Based on their research, we aim to improve the policy in two directions: first, the subsidy rate can be designed to respond to the economic variable which is easy to observe in practise. Second, we would like to investigate whether we can find a welfare-improving macroprudential policy.

This paper provides an improved version of macroprudential policy which performs better than the GKQ policy. The policy framework is similar: the central bank subsidizes the bank equities by taxing the bank assets. The difference is that the central bank chooses the bank equity subsidy according to the bank external capital ratio in the alternative policy. The alternative policy has two advantages. First, the bank capital ratio is an observable target for the central bank and the capital ratio reveals banks' ability to hedge financial risks. Second, the alternative policy is more welfare improving.

We compare the performance of the alternative macroprudential policy to the GKQ policy and find the following results: first, the new policy helps reduce the volatilities of both risky returns and the credit spread, so it helps enhance the stability of the financial system and mitigate the financial frictions. Second, given the same level of subsidy per unit of bank equity in the policy framework, the new policy creates a lower moral hazard cost in the banking sector. Third, the new policy reduces the bank's cost for issuing bank equities and deposits, which decreases the return per unit of capital and raises the firm's borrowing. According to the diminishing return of capital in the production function, a lower level of return on capital implies a higher capital stock and a higher level of output and consumption. We compare welfare under different policy scenarios using consumption equivalents and show that the new policy provides higher welfare. The result is robust to the choice of parameters and the size of the financial shock.

The optimal parameters of the alternative policy show that the central bank should offer a progressive bank equity subsidy with respect to the aggregate bank capital ratio. To be specific, the central bank gives a high subsidy at normal times to keep the bank capital ratio high. After the financial shock has occurred, bank assets suddenly deteriorate, thus reducing the bank capital ratio. In that case, the central bank lowers the subsidy so it allows the bank's capital ratio to be low for a longer period of time. The macroprudential

[2]Time-varying capital requirement and countercyclical capital buffers are analyzed in Kashyap and Stein (2004) and Dewatripont and Tirole (2012). A substantially higher capital ratio than the market determined ratio is proposed in Admati et al. (2010) and Hanson et al. (2011). Perotti and Suarez (2009) propose liquidity risk charges to encourage banks to issue more bank equities, and it can overcome the procyclicality of macroprudential policy.

policy effect is consistent with the countercyclical capital ratio requirement in Kashyap and Stein (2004) and Hanson et al. (2011). They propose a high capital ratio requirement before the financial crisis and lowers the requirement in hard times because it is more difficult to retain bank capital just after the financial crisis. Moreover, the new policy is in line with the macroprudential capital policy tool from Bank of England in Harimohan and Nelson (2012).

The outline of the paper is as follows. Section 2 introduces the model. Section 3 provides the simulation results, Section 4 compares the performances of different policies and Section 5 concludes the paper. We provide detailed derivations of the model in the Appendix.

2 The Model

In this section, we describe four sectors in the model: households, goods producers (firms), capital producers and banks. The framework is from GKQ. It is a DSGE model with financial frictions. In the model, individual banks have the alternatives to issue either short-term debts or bank equities. This feature helps explain why banks choose a low capital ratio in equilibrium. The financial friction is from the agent problem between banks and bank managers. Briefly, the moral hazard problem alters banks' balance sheet decisions, and it lowers the bank capital ratio on the balance sheet. The summary of the system of equations is shown in Appendix 6.1.

It is assumed that capital producers endure flow-variable adjustment costs, so the resource constraint is

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{1}$$

where Y_t is domestic final output, C_t is consumption, I_t is investment and $f(\cdot)$ are adjustment costs. The function of adjustment costs is convex with $f(1) = f'(1) = 0$ and $f''(x) > 0$ for any $x > 0$.[3]

Following GKQ, the financial crisis is modelled as a negative exogenous shock to the aggregate physical capital stock. As will become clear, a deterioration of the capital stock will be reflected in the asset side of individual banks. This is how the asset price variation is introduced,

$$K_{t+1} = \psi_{t+1} S_t,^{4} \tag{2}$$

where K_{t+1} is the aggregate capital stock at the beginning of time $t+1$, S_{t+1} is the accumulated aggregate capital at the end of time t and ψ_{t+1} is the capital quality shock from time t to $t+1$. The shock ψ_{t+1} (> 0) is an i.i.d. process with the unconditional mean 1.

[3]In the simulations, f is specified as a quadratic function: $f(\frac{I_\tau}{I_{\tau-1}}) = \Psi\left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2$.

[4]For the purpose of tractability, we follow GKQ and consider only the capital quality shock without incorporating other important shocks, such as TFP shock. Therefore the model may not capture standard business cycle moments. I acknowledge that adding those shocks will affect the stochastic steady state values and the welfare calculation. Results are not tested along this dimension.

Non-financial firms accumulate capital and produce final output at a constant physical depreciation rate, δ. The aggregate physical capital accumulation is subject to

$$S_t = (1-\delta)K_t + I_t. \tag{3}$$

After the realization of the capital quality shock, ψ_{t+1} from time period t to $t+1$, capital 'in progress' (S_t) is transformed into the realized capital stock (K_{t+1}) at time $t+1$.

2.1 Households

A representative household has a continuum of members with mass 1. There are f fraction of workers and $1-f$ fraction of bankers in the household. Bankers work as bank managers and make decisions on the bank balance sheet. When they are forced to quit the banking sector to become workers, they transfer the net worth in banks back to the household as dividends. The probability of bankers quitting in every period is σ. Workers rent labour to firms to gain wages, and they may become new bankers with probability $\frac{(1-\sigma)(1-f)}{f}$ each period. In this way, the fraction of bankers to workers remains constant.

The utility function follows Guvenen (2009) and Greenwood, Hercowitz and Huffman (1988). The reason for this form of preference is: first, there is no wealth effect on labour supply. Second, it produces labour volatility with little cost of complexity. Third, the habit formation improves the quantitative performance of the model. The household has an expected discounted lifetime utility

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau, C_{\tau-1}, L_\tau\right) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} \left(C_\tau - hC_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi}\right)^{1-\gamma}, \tag{4}$$

where $\mathbb{E}_t(\cdot)$ denotes the expectations conditional on the information set at time t, L_t is the labour input into production, β is the time discount factor, γ is risk aversion, h is the habit parameter, χ is the weight parameter towards labour and φ is the inverse Frisch labour elasticity.

It is assumed that households do not provide funds directly to firms. They only lend money to banks. Households can choose to buy either risk-free return debts (deposits, D_t) or bank equities (e_t). The representative household maximizes his or her expected utility by choosing consumption, labour supply, deposits and bank equities (C_t, L_t, D_t, e_t) subject to the budget constraint:

$$C_t + D_t + q_t e_t = W_t L_t + \Pi_t + R_t D_{t-1} + R_{e,t} q_{t-1} e_{t-1}, \tag{5}$$

where W_t is wage, Π_t is the net profit from capital producers plus banker's final dividend payments, q_t is the price of bank equity, R_t is the risk-free return on deposits and $R_{e,t}$ is the return on bank equities from time $t-1$ to t. Denote $U_{C,t}$ as the marginal utility to consume at time t and $\Lambda_{t,\tau}$ as the stochastic discount factor from t to τ:

$$U_{C,t} \equiv \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi}\right)^{-\gamma} - \beta h \left(C_{t+1} - hC_t - \frac{\chi}{1+\varphi} L_{t+1}^{1+\varphi}\right)^{-\gamma}, \tag{6}$$

$$\Lambda_{t,\tau} \equiv \beta^{\tau-t}\frac{U_{C,\tau}}{U_{C,t}}. \tag{7}$$

By solving the representative household's optimization problem, we derive two Euler equations because households can choose either bank deposits or bank equities,

$$R_{t+1}\mathbb{E}_t\left(\Lambda_{t,t+1}\right) = 1, \tag{8}$$

$$\mathbb{E}_t\left(\Lambda_{t,t+1}R_{e,t+1}\right) = 1, \tag{9}$$

and we can derive the first order condition for labour supply,

$$\mathbb{E}_t U_{Ct} W_t = \chi\left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi-\gamma}\right)L_t^{\varphi}. \tag{10}$$

2.2 Goods Producers

Competitive goods producers produce identical final output using capital and labour. It is assumed that they have to borrow funds from banks by issuing firm securities. Moreover, banks have access to all information from the borrowing firms. So they can borrow without any frictions by committing to pay the entire profit to banks via firm equities. Firms produce output with a constant returns to scale Cobb-Douglas production function,

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},\ 0 < \alpha < 1 \tag{11}$$

where A_t denotes aggregate productivity. This paper focuses on investigating the effect of the capital quality shock ψ_t, which captures the financial crisis. For simplicity, we assume that $A_t = 1$. The first order condition with respect to labour is

$$W_t = (1-\alpha)\frac{Y_t}{L_t}.$$

Let Z_t denote the gross return on per unit of capital before the capital quality shock,

$$Z_t = \frac{Y_t - W_t L_t}{K_t} = \alpha\frac{Y_t}{K_t} = \alpha\frac{AL_t^{1-\alpha}}{K_t^{1-\alpha}} = \alpha A\left(\frac{L_t}{K_t}\right)^{1-\alpha}.$$

It it assumed that firms normalize firm security to correspond to one unit of capital, so Z_t also stands for the return on firm equity. Since all profit flows go from firms to banks via the firm equities, manufacturers have no profit left in each period. The return on firm equity consists of the gross return of capital, physical depreciation, and the change in the capital price,

$$R_{k,t} = \frac{\left[Z_t + (1-\delta)Q_t\right]\psi_t}{Q_{t-1}}. \tag{12}$$

Similarly, the return on bank equity[5] is

$$R_{e,t} = \frac{\left(Z_t + (1-\delta)q_t\right)\psi_t}{q_{t-1}}. \tag{13}$$

[5]Banks normalize the value of bank equity to make their claim correspond to the return of the bank asset, so as to relate it to the gross return of capital. After the normalizations of both firm equities and bank equities, the price of capital equals the price of the firm security, Q_t.

2.3 Capital Producers

Capital producers use final output to produce capital with flow-variable adjustment costs. They transfer their profit back to households in each period because households have the ownership. The capital producers choose the investment I_t to produce capital given the capital price Q_t subject to the adjustment costs,

$$\max \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau I_\tau - I_\tau \left[1 + f(\frac{I_\tau}{I_{\tau-1}}) \right] \right\}.$$

The concavity of the objective function ensures that the interior solution exists. The solution shows that the marginal cost of capital production equals the price of capital Q_t.

$$Q_t = 1 + f(\frac{I_t}{I_{t-1}}) + \frac{I_t}{I_{t-1}} f'(\frac{I_t}{I_{t-1}}) - \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 f'(\frac{I_{t+1}}{I_t}) \right] \tag{14}$$

2.4 Banks

Individual banks work as financial intermediaries. They issue outside bank equities and take deposits from households and lend them to firms. Banks keep their profit as net worth and face the balance sheet constraint in each period, where the total value of loans cannot exceed their raised funds plus the net worth.

$$Q_t s_t \leq n_t + q_t e_t + d_t, \tag{15}$$

where s_t are firm securities that the individual banks buy from the goods producers, e_t are bank outside equities, n_t is net worth. d_t are bank deposits and q_t is the price of bank equity. As mentioned in Section 2.2, the gross return of physical capital flows from firms to banks through firm equities. It means that any factor that changes the return of capital will also cause variation in bank assets.

In GKQ, the following two assumptions ensure that banks cannot accumulate enough net worth to avoid borrowing from households. First, new bankers do not have sufficient net worth when they go into the banking sector. Second, banks have a constant probability of quitting the banking sector in every period. Given the above restrictions, banks need to issue equities and short-term debts in each period. A law of motion of individual banks' net worth is

$$n_t = R_{k,t} Q_{t-1} s_{t-1} - R_t d_{t-1} - R_{e,t} q_{t-1} e_{t-1}. \tag{16}$$

Before introducing the moral hazard problem, we define two ratios in the banking sector: the leverage ratio ϕ_t and the outside equity ratio x_t.

$$\phi_t = \frac{Q_t s_t}{n_t}, \tag{17}$$

$$x_t = \frac{q_t e_t}{Q_t s_t}. \tag{18}$$

Leverage is the ratio of bank assets to net worth. The bank outside equity ratio is the ratio of bank outside equities to bank assets. The fraction of outside equities plays an important role for the financial stability in this model. While banks are not able to accumulate enough net worth, they can always choose a higher outside equity ratio to hedge the financial risks. Although the bank capital ratio also includes inside equities in common usage, we use the term *bank capital ratio* to refer to the bank outside equity ratio, x_t, and they are interchangeable hereafter.

The objective of the individual banks is to accumulate the net worth and get the maximal dividends when they are forced to quit the banking sector. They maximize the expected discounted future dividend payments. Letting σ denote the banker's probability of exiting the financial market, the value function is

$$V_t(n_t) = \max \mathbb{E}_t \left[\sum_{\tau=t+1}^{\infty} (1-\sigma)\sigma^{\tau-t-1} \Lambda_{t,\tau} n_\tau \right]. \tag{19}$$

A moral hazard problem is embedded between shareholders and bank managers as in Gertler and Karadi (2010). After banks have obtained funds from households, the bank manager has incentives to transfer a fraction of the bank assets (Θ_t) back to his family. As households recognize this, they limit the funds provided to banks and change the proportion of short-term debts and bank equities.

The fraction of diversion from bank assets depends on the current composition of the bank's borrowing, x_t. At the margin, it is more difficult to divert the bank assets funded by short-term debts than outside equities. This is due to the fact that short-term debts have the first priority and require banks to meet a fixed amount of payment with full responsibility. In contrast, bank equity payment is time-contingent and related to the return of bank assets, which is more difficult to monitor by bank outside equity holders. This idea comes from Calomiris and Kahn(1991). Assume that the fraction of diversion Θ_t is a function of the outside equity ratio, which is specified as quadratic:

$$\Theta(x_t) = \theta \left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2 \right). \tag{20}$$

where $\varepsilon < 0$, $\kappa > 0$. Households get some advantages in monitoring the bank managers by having at least some bank equities ($\varepsilon < 0$). Moreover, we restrict the marginal fraction of diversion $\theta\left(\varepsilon + \kappa x_t\right)$ to be positive because bank managers can divert more funds with a higher fraction of equities on the balance sheet.

If the bank manager diverts a fraction of bank assets back to the household, the bank defaults. In this case, the bank faces bankruptcy, the creditors only get $1 - \Theta(x_t)$ fraction of assets as liquidation and the bank manager gets $\Theta(x_t)$ fraction of assets without dividends. Since households recognize the bank managers' incentive to divert funds, they restrict their lending to ensure that the bank manager's payoff from diverting funds does not exceed the expected dividends at each date. By doing this, the bank managers will not choose to divert funds, but the banks face the following borrowing constraint

$$V_t(n_t) \geq \Theta(x_t) Q_t s_t. \tag{21}$$

The moral hazard constraint involves the bank's value function. We find the closed form of V_t as follows.

$$V_t(n_t) = (\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t, \tag{22}$$

where the coefficients are recursively defined by

$$v_t \equiv \mathbb{E}_t\left(\Lambda_{t,t+1}\Omega_{t+1}\right) R_{t+1}, \tag{23}$$

$$\mu_{s,t} \equiv \mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\right], \tag{24}$$

$$\mu_{e,t} \equiv \mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}\left(R_{t+1} - R_{e,t+1}\right)\right], \tag{25}$$

$$\Omega_{t+1} \equiv (1-\sigma) + \sigma\left[\phi_t\left(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1}\right) + v_{t+1}\right]. \tag{26}$$

The optimal bank leverage ratio is

$$\phi_t = \frac{v_t}{\Theta(x_t) - \left(\mu_{s,t} + x_t\mu_{e,t}\right)} \tag{27}$$

The derivation method is shown in Appendix 6.4.

Here Ω_{t+1} is the shadow price of the net worth tomorrow, so $\Lambda_{t+1}\Omega_{t+1}$ is the discounted shadow price of net worth. v_t is the cost of banks issuing deposits that can be saved from an additional net worth tomorrow. In Eq. (25), $\mu_{e,t}$ is the extra cost of issuing bank deposits rather than bank equities from the additional future net worth. $\mu_{e,t}$ is positive in the stochastic steady state. This means that the cost of issuing bank equity is lower from the perspective of the banks. In Eq. (24), $\mu_{s,t}$ is the net profit from the additional future net worth if banks issue bank deposits. By Eqs. (24) and (25), $\mu_{s,t} + x_t\mu_{e,t}$ is the net profit of the bank asset with the balance sheet x_t,

$$\mu_{s,t} + x_t\mu_{e,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}R_{k,t+1} - (1-x_t)\Lambda_{t+1}\Omega_{t+1}R_{t+1} - x_t\Lambda_{t+1}\Omega_{t+1}R_{e,t+1}\right]. \tag{28}$$

If we consider the evolution of aggregate net worth in the banking sector, the dynamics of new bankers and old bankers should be taken into account,

$$N_t = \sigma\left(R_{t,k}Q_{t-1}S_{t-1} - R_t D_{t-1} - R_{e,t}q_{t-1}E_{t-1}\right) + N_{y,t}.^{6} \tag{29}$$

The initial wealth of new bankers ($N_{y,t}$) is assumed to be a fixed fraction of bank assets (ξ) at the end of the current period,

$$N_{y,t} = \xi R_{k,t}Q_{t-1}S_{t-1}.$$

[6] We use lower case notations for individual variables and capital letters for aggregate variables.

2.5 Macroprudential Policy

Because of the financial frictions, individual banks tend to issue more short-term debts than in a frictionless financial market. This makes banks exposed to risks because they do not consider the externality that holding more short-term debts increases the systemic risk and makes the financial system more vulnerable. To overcome this problem, a macroprudential policy is introduced in GKQ. The policy induces individual banks to issue more bank equities instead of short-term debts, so as to raise the bank capital ratio. Since the return on bank equity ($R_{e,t}$) is time-contingent and it is linked to the performance of the bank's investment projects, the bank risk can be better hedged with a higher bank equity ratio. In the macroprudential policy framework, the central bank taxes private intermediated bank assets and offers a subsidy to bank equities. Thus, the bank's balance sheet constraint is modified into

$$\left(1+\tau_t^k\right) Q_t s_t = n_t + \left(1+\tau_t^e\right) q_t e_t + d_t, \tag{30}$$

where τ_t^k is the tax rate on bank assets and τ_t^e is the subsidy rate on bank equities. The tax and the subsidy level (τ_t^k and τ_t^e) are determined by the macroprudential policy rule. To ensure that the policy is fiscal neutral on the balance sheet, we have the following condition,

$$\tau_t^k = x_t \tau_t^e.$$

In GKQ, the central bank chooses a bank equity subsidy depending on the shadow cost of the bank deposit, v_t,

$$\tau_t^e = \frac{\tau_1}{v_t}, \tag{31}$$

where τ_1 is the policy parameter chosen by the central bank. From Eq. (23), v_t is the bank's cost of issuing short-term debts from the additional net worth tomorrow. The intuition of the GKQ policy is: if v_t is low, banks tend to issue more deposits, so that the central bank subsidizes more on bank equities. In this policy framework, the bank equity subsidy raises the extra cost of banks issuing a deposit relative to bank equity from $\mu_{e,t}$ to $\mu_{e,t} + \tau_1$. From the perspective of the bank, it costs less to issue bank equities than before. Therefore, banks tend to issue more bank equities and the bank capital ratio will increase. However, it has not been shown that this policy rule is optimal for improving welfare. The motivation for the policy modification is: first, while GKQ take into account the policy effect on the bank's cost for issuing deposit in their policy, they do not consider the effect on the profit of the bank asset. Second, the shadow cost of the deposit is not an easily observed target for the central bank in practice.

Here we introduce an alternative macroprudential policy and name it *capital ratio policy*: the central bank chooses the bank equity subsidy to react to the *aggregate* bank capital ratio, x_t. We introduce this new policy because a number of studies in the literature such as Perotti and Suarez (2009) and Hanson et al. (2011) suggest that macroprudential policy should focus on regulations of bank capital and capital requirements. The policy rule is

$$\tau_t^e = \tau_0 x_t + A_{\tau 0},^7 \tag{32}$$

where $A_{\tau 0}$ is the level parameter of subsidy and τ_0 is the ratio parameter. Both parameters are chosen by the central bank, and they can be negative. When τ_0 is positive, the central bank offers a progressive subsidy rather than a regressive subsidy. In the capital ratio policy, $\tau_0 x_t$ depends on the path of the bank's capital ratio after a financial shock and $A_{\tau 0}$ is the subsidy level exogenously chosen by the central bank.

With the capital ratio policy, individual banks' value function is modified from (22) to

$$V_t(n_t) = \left[\left(\mu_{s,t} - \tau_t^k v_t\right) + \left(\mu_{e,t} + \tau_t^e v_t\right) x_t\right] Q_t s_t + v_t n_t. \tag{33}$$

The bank's optimal choice of capital ratio is also different:

$$\left(\mu_{s,t} + \mu_{e,t} x_t\right)\left(\varepsilon + \kappa x_t\right) = \left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2\right)\left[\mu_{e,t} + v_t\left(\tau_0 x_t + A_{\tau 0}\right)\right]. \tag{34}$$

The derivation is similar to that of the baseline model without policy.

The new policy focuses on the bank's capital ratio, x_t, which is important information for the bank's ability to hedge against financial risks. The results show that the capital ratio policy helps banks keep the high capital ratio before the financial shock and allows for a lower capital ratio after the shock. The implementation of the new policy is consistent with the time-varying capital ratio requirement. In the simulation part, we compare the policy performances and explain why the new policy provides a higher welfare than the GKQ policy.

Before the simulation part, we describe how we measure welfare in the model. Welfare is the unconditional stochastic steady state value of households' lifetime utility,

$$\mathcal{W}_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau, L_\tau\right). \tag{35}$$

We take the second order approximation of the welfare around the stochastic steady state, so that volatility is included. We compare welfare under different policy scenarios using consumption equivalents. More details of the welfare description are shown in the Appendix 6.6.

3 Simulation Results

In this section, we provide the simulation results to show how the capital ratio policy mitigates the impact of the financial shock and improves welfare. Simulations of the baseline model without policy and the GKQ policy are also provided for comparison.

To capture the risk perceptions channel in the model, we use stochastic steady state following the literature Campbell (1994), Lettau (2003) and Coeurdacier et al. (2011). The

[7] Here x_t is an aggregate level of bank capital ratio so that one individual bank cannot affect the tax rate τ_t^e. In equilibrium, all individual banks choose the same level of x_t.

stochastic steady state differs from non-stochastic state in the aspect of second moments. These second moments of variables have an impact on banks' balance sheet decisions and affect the steady state.

The computational method in this paper follows de Groot (2013). We do the second order approximation of the model around the deterministic steady state and use the iteration method to get the stochastic steady state. We find the stochastic steady state of our model such that the second moments generated by this stochastic steady state lead back to the same stochastic steady state.

There are 15 parameters in the baseline model. The parameter values are from GKQ and they are summarized in Table 1.

Table 1: Parameters

β	0.99	discount factor
γ	2	relative risk aversion parameter
h	0.75	habit formation coefficient
χ	0.25	weight coefficient of labour
φ	0.33	inverse Frisch elasticity of labour supply
α	0.33	share of capital
δ	0.0025	physical depreciation rate
Ψ	1	the elasticity of the price of capital to investment
σ	0.9685	bank probability of survival
ξ	0.00289	households transfer ratio to new banks
θ	0.264	moral hazard parameter
ε	-1.21	bank's diversion parameter (linear term)
κ	13.41	bank's diversion parameter (quadratic term)
$E(\psi_t)$	1	mean of the financial shock
$std(\psi_t)$	0.69%	standard deviation of the financial shock

In both macroprudential policies, the central bank taxes bank assets and subsidizes bank equities (Eq. (30)). The difference is: in the GKQ policy, they choose the bank equity subsidy to respond to the shadow cost of the deposit (Eq. (31)); in the capital ratio policy, the central bank responds to the aggregate bank capital ratio level (Eq. (32)).

The stochastic steady states of key variables are shown in Table 2. The first two columns are stochastic steady-state values and standard deviations in the baseline model without policy. As compared to the no policy scenario, the capital ratio policy increases x_t to 15.56% in steady state, but it is lower than that in the GKQ policy (16.19%). Therefore, the new policy also works as a capital ratio requirement. A lower steady-state value of the bank capital ratio in the capital ratio policy reduces the bank manager's fraction of diversion from banks (Eq. (20)). In this respect, the new policy causes a lower moral hazard cost than the GKQ policy.

The capital ratio policy results in a higher steady-state value of the net profit of bank assets, $\mu_{s,t} + x_t\mu_{e,t}$, which makes it more profitable for bank managers to keep bank assets in the bank rather than diverting them. At the same time, the lower volatility of $\mu_{s,t} + x_t\mu_{e,t}$ enhances the stability of the financial sector from the perspective of the bank.

In the capital ratio policy, banks have a higher leverage ratio due to a higher shadow cost of deposit, a lower fraction of diversion and a higher $\mu_{s,t} + x_t\mu_{e,t}$ (Eq. (27)). Thus, banks have a greater ability to absorb funds for a given amount of net worth, N_t. This creates a higher value of bank assets and firm borrowing.

Table 2: Risky Steady States[8]

		No Policy		GKQ Policy		New policy	
		Steady state	Std. Dev.	Steady state	Std. Dev.	Steady state	Std. Dev.
Output	Y	23.58	0.9526	24.17	0.9573	24.37	0.9334
Consumption	C	18.42	0.7165	18.82	0.7147	18.95	0.6974
Labour	L	8.10	0.2414	8.25	0.2390	8.30	0.2266
Capital	K	206.29	12.7281	214.25	12.9550	216.91	12.4599
Net Worth	N	32.48	4.9305	31.84	6.1674	31.33	7.4127
Risk-free Return	r (%)	1.02	0.0017	0.985	0.0021	0.974	0.0022
Risky Return	r_k (%)	1.27	0.0141	1.227	0.0108	1.211	0.0102
Credit Spread	$r_{k,t}$ -r_t (%)	0.25	0.0155	0.24	0.0125	0.237	0.0119
Capital ratio	x	0.1036	0.0372	0.1619	0.0489	0.1556	0.1130
Deposit cost	v	1.6051	0.1284	1.7152	0.1552	1.7546	0.1377
Excess Equity Cost	μ_e	0.047	0.0018	0.030	0.0042	0.017	0.0054
Bank asset profit	μ_s	0.2447	0.6862	0.3109	0.1981	0.3187	0.0827
Overall profit	$\mu_s + x\mu_e$	0.2496	0.6843	0.3157	0.1960	0.3213	0.0799
Leverage ratio	ϕ	6.4876	0.7195	6.7123	0.9813	6.9094	1.2748
Subsidy	τ^e	0	N/A	0.0016	1.45×10^{-4}	0.00155	2.36×10^{-4}
Tax	τ^k	0	N/A	2.6×10^{-4}	1.02×10^{-4}	2.4×10^{-4}	5.39×10^{-4}
Consumption equivalent	Γ (%)	0	N/A	0.5264	N/A	1.6184	N/A

In addition, the steady state of risky return ($R_{k,t}$[9]) and credit spread ($R_{k,t} - R_t$) are reduced more and their volatilities are lower in the capital ratio policy. A lower steady-state value and volatility of the credit spread suggest that the financial market is more stable. According to the diminishing return of capital in the Cobb-Douglas function, the lower steady state of return on the bank asset implies a larger physical capital stock, which leads to a higher level of output and consumption. The level of $R_{k,t}$ is lower in the capital ratio policy. That is to say, the capital ratio policy performs better at raising total output and consumption.

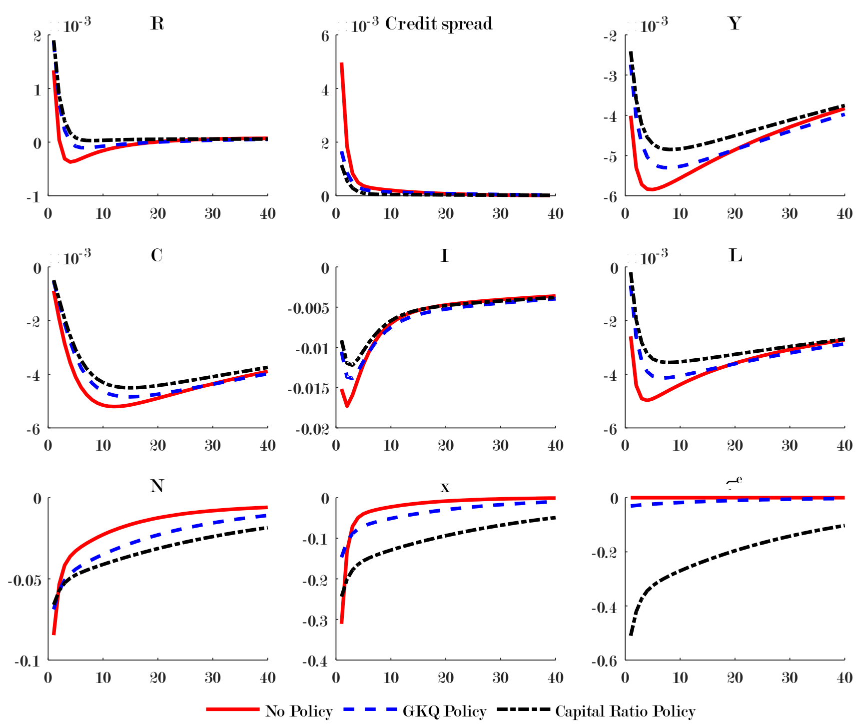
[8]In Table 2, the policy parameters of both the GKQ policy and the capital ratio policy are optimized. In the GKQ policy, the optimal parameter τ_1^* is 0.0028 (see Firuge 13 in the Appendix 6.2); in the capital ratio policy, the optimal parameters $(\tau_0^*, A_{\tau 0}^*)$ are $(0.0209, -0.0017)$. The way of finding optimal parameters is provided in Section 4.2.

[9]All returns in Table 2 are quarterly interest rates.

As a result, the capital ratio policy improves the welfare closer to the economy without financial frictions. We can derive the equilibrium of a frictionless economy by solving the social planner's problem, choosing $(Y_t, L_t, C_t, I_t, S_t)$ to maximize the representative household's utility subject to the resource constraints (1), (2), (3) and (11). Compared to the baseline model, the frictionless economy has a consumption equivalent of 4.8 percent. Table 2 shows that the capital ratio policy can achieve a higher welfare with consumption equivalents of 1.62 percent than 0.53 percent in the GKQ policy.

Figure 1 illustrates the impulse responses of important variables after the financial shock. The financial shock is a decrease of ψ_t by one unit of its standard deviation. All variables are in log deviations from steady state except the credit spread and the credit spread is presented as the actual deviation. In general, we can see that the capital ratio policy has a better stabilization effect on economic variables than the GKQ policy. It is consistent with the smaller second moments of variables in the capital ratio policy in Table 2.

Figure 1: Baseline Model v.s. Two Policies



When the economy is hit by a financial shock, the credit spread increases by 50 basis points without the policy. The GKQ policy could mitigate the sudden change of the credit spread down to 20 basis points, while the capital ratio policy decreases the credit spread to only 10 basis points. It shows that the stability of the financial market is improved more by the capital ratio policy. The financial shock to the bank's balance sheet is transmitted to the real economy through the sudden increase in the credit spread after the financial shock. Since the sudden increase in the credit spread is smaller under the capital ratio policy than under the GKQ policy, the cost of investment is increased by a smaller amount. Therefore, we have a greater stabilization both for financial sector variables and for real variables (e.g., output and consumption) in the economy.

As compared to the GKQ policy, the way in which the central bank operates the bank equity subsidy (τ_t^e) over time is quite different in the capital ratio policy. In the GKQ policy, the central bank offers a subsidy to the bank equity of 16 basis points quarterly in a steady state. When there is a financial shock, there is an increase in the shadow cost of the bank deposit, so the central bank decreases the subsidy. Since the sensitivity of v_t to the capital quality shock is small, the subsidy only decreases by 4%. In the capital ratio policy, the bank equity subsidy reacts directly to the current bank capital ratio, x_t. The central bank offers a 15.5 basis point subsidy per unit of bank equity before the financial shock. As the capital ratio suddenly decreases by 30% after the shock, the bank equity subsidy decreases by almost 50%. That reduction of the subsidy keeps the bank capital ratio at 12% over time, around 4% lower than before the financial shock. It means that the capital ratio policy has a larger countercyclical capital ratio requirement than the GKQ policy. The central bank requires a high capital ratio before the financial shock, but allows individual banks to keep a lower capital ratio for a longer period after the shock. The new policy is consistent with the literature on a time-varying capital ratio requirement, e.g. Kashyap and Stein (2004) and Hanson et al. (2011). And the capital ratio policy is in line with the macroprudential capital policy tool of Bank of England (see Harimohan and Nelson (2012)). We will discuss the underlying mechanisms of different macroprudential policies in the next section.

The result in the simulation is robust to the choice of parameters in real sectors as well as the size of the financial shock (a standard deviation from 0.5% to 2%). It is also robust to the parameter choice in the financial sector within a reasonable interval. To be specific, the results hold when κ ranges from 5 to 20, ε from -4 to 0 and θ from 0.15 to 0.35.

4 Different Channels of Macroprudential Policy

In this section, we provide an explanation for why the capital ratio policy gives higher welfare than the GKQ policy. We find three different channels through which welfare is affected.

The first channel is the *balance sheet channel*: it works through the bank equity subsidy (τ_t^e) in the balance sheet (Eq. (30)). When the central bank raises τ^e, the individual banks tend to issue more bank equities and less deposits. This helps individual banks hedge the financial risks. Moreover, from the perspective of the bank, the cost for issuing equities is lower than for deposits ($\mu_{e,t}$ is positive in (25)). As the bank capital ratio goes up, the policy reduces the bank's cost for borrowing funds from households.

The second channel is the *moral hazard channel:* as macroprudential policies raise the capital ratio, the bank manager's fraction of diversion increases (in Eq. (20)). A higher capital ratio raises the moral hazard cost and reduces the welfare.

The third channel is the *cost reduction channel.* From the perspective of the individual bank, there is a change in the cost of issuing funds and the profit of the bank asset as the central bank changes the bank equity subsidy level. This channel includes the general equilibrium effect on the shadow cost of the deposit and the bank equity as well as the shadow profit of the bank asset (Eqs. (23), (24) and (25)). When the bank equity subsidy is the same among different policies, the cost reduction effect in the GKQ policy and the capital ratio policy is still different.

In the following analysis, we change policy parameters to shut down the balance sheet channel among different policy scenarios, and investigate how the moral hazard channel and the cost reduction channel affect welfare. Our analysis evaluates the key variables in a stochastic steady state, and investigate the policy effect on welfare from the general equilibrium perspective. The results show that the capital ratio policy gives higher welfare than the GKQ policy because the cost reduction channel performs better.

4.1 Moral Hazard Channel v.s. Cost Reduction Channel

Recalling the policy rules, (31) is for the GKQ policy and (32) is for the capital ratio policy. For each policy rule of τ_t^e, the bank's optimization problem is different. We extract the differences from the system of equations, and the following Eqs. (36) and (37) are optimal choices of x_t with the GKQ policy and the capital ratio policy. To make the presentation clear, we change notations for different policies where the variables in the GKQ policy have tilde.

$$\left(\widetilde{\mu}_{s,t} + \widetilde{\mu}_{e,t}\widetilde{x}_t\right)(\varepsilon + \kappa\widetilde{x}_t) = \left(1 + \varepsilon\widetilde{x}_t + \frac{\kappa}{2}\widetilde{x}_t^2\right)\left[\widetilde{\mu}_{e,t} + \tau_1\right]. \tag{36}$$

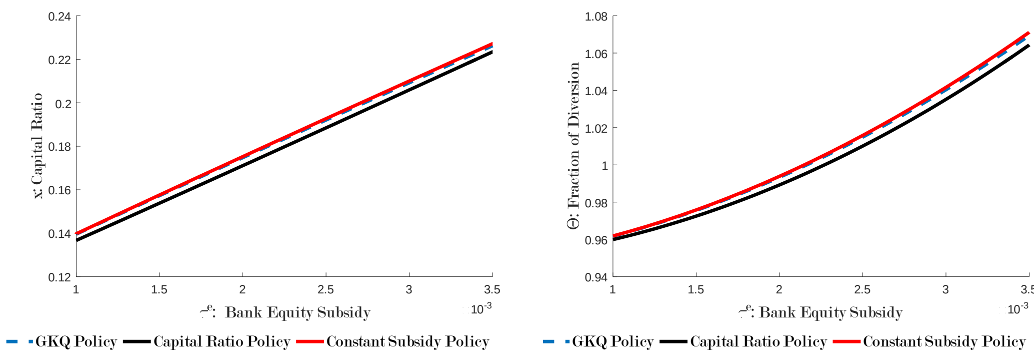
$$\left(\mu_{s,t} + \mu_{e,t}x_t\right)(\varepsilon + \kappa x_t) = \left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right)\left[\mu_{e,t} + \upsilon_t\tau_t^e\right]. \tag{37}$$

If the policy were only transmitted through the balance sheet channel, we would have the same effect on welfare by choosing the same level of τ_t^e among different policy scenarios. However, in the capital ratio policy, τ_t^e is a function of the aggregate capital ratio and it has a general equilibrium effect on the bank's optimal choice of x_t. So there are two other channels that perform differently under different macroprudential policies.

To analyze how the moral hazard channel and the cost reduction channel affect other variables and welfare, we fix the subsidy per unit of bank equity τ^e in (30) across different policies to shut down the balance sheet channel. To check the sensitivities of other key variables to the bank equity subsidy under different policy rules, we vary τ^e by choosing different policy parameters. In the new policy, the central bank can choose two parameters $(\tau_0, A_{\tau 0})$. In this case, we fix one parameter $A_{\tau 0} = -0.0017$ where the highest welfare can be achieved, and then vary τ^e by choosing different values of the other parameter τ_0. Here, we introduce a *constant subsidy policy* as a benchmark: the central bank provides a constant subsidy per unit of bank equity ($\tau_t^e = \tau$). In this policy scenario, we can see the macroprudential policy effect that only goes through the balance sheet channel.

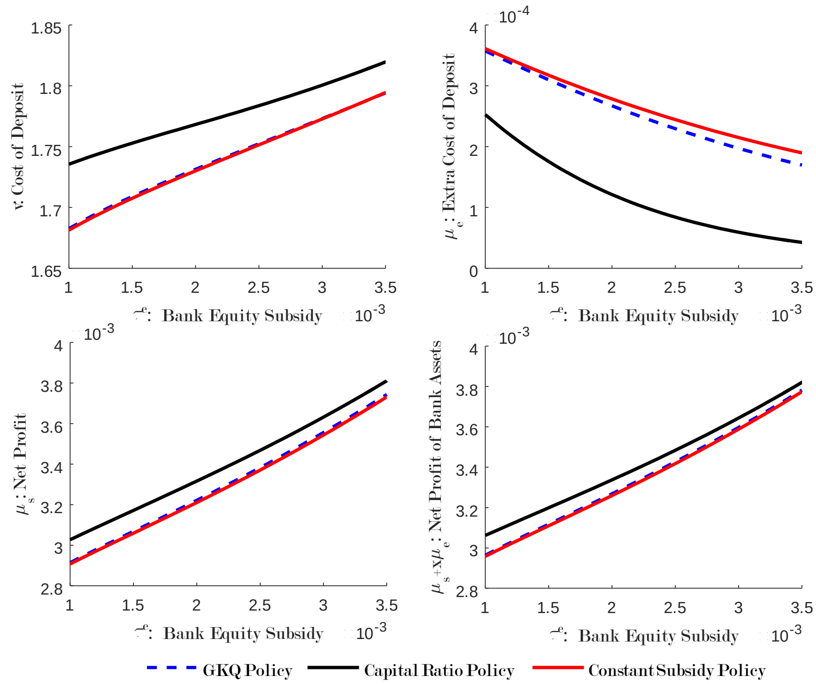
Since those first affected by the policy are banks, we start by checking the key variables in the banking sector (e.g. the capital ratio (x), the shadow cost of the deposit (υ) and the net profit of the bank asset ($\mu_s + x\mu_e$)). As the central bank changes the bank equity subsidy in three different policies, the features of the variables in the constant subsidy policy and the GKQ policy are very similar in Figures 2, 3 and 4. This implies that the GKQ policy effect mostly goes through the balance sheet channel. The path of a shadow cost of deposit does not have large impact on welfare because its sensitivity to τ^e is relatively small in steady state. To be specific, as the central bank varies the bank equity subsidy from 5 to 40 basis points, the shadow cost of the deposit only changes from 1.65 to 1.8, but the bank capital ratio increases from 12% to 25%.

Figure 2: Capital Ratio x and Moral Hazard Cost Θ



As we can see from Figure 2, the capital ratio goes up as the central bank raises τ^e for all three policies. The policy effect of τ^e on the capital ratio is very similar between the GKQ policy and the constant subsidy policy, but it is lower in the capital ratio policy. This is due to the fact that the cost savings when the bank switches from bank deposits to bank equities are lower in the capital ratio policy (see μ_e in the top-right panel of Figure 3), so the bank has less incentive to raise the capital ratio. Given any level of τ^e, a lower capital ratio level under the capital ratio policy leads to a smaller diversion, Θ. Therefore, the moral hazard cost is lower.

Figure 3: Variables in the Banking Sector

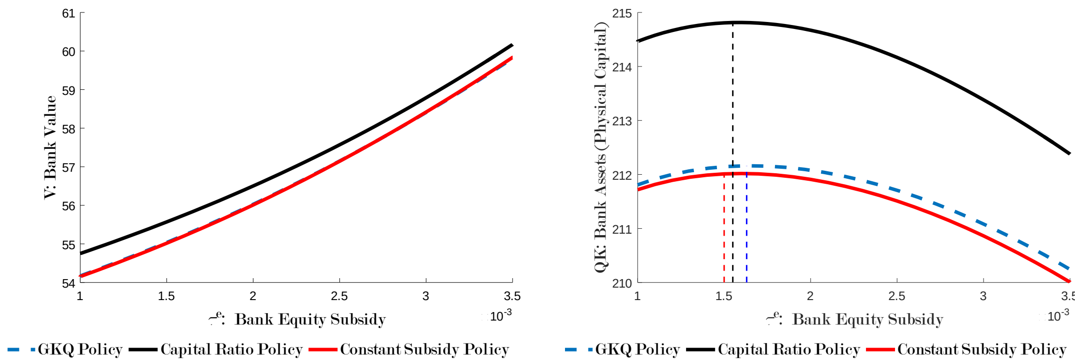


As shown by the bottom-right panel of Figure 3, the net profit of bank assets from an additional net worth tomorrow, $\mu_s + x\mu_e$, is higher under the capital ratio policy because of a higher value of μ_s. A higher level of $\mu_s + x\mu_e$ makes each unit of net worth more valuable in the future.

The results show that the capital ratio policy leads to a lower fraction of diversion (Θ) and a higher net profit of the bank asset ($\mu_s + x\mu_e$), both of which affect the value of individual banks, V. On the one hand, a higher net profit of the bank asset increases the bank value (Eq. (22)). On the other hand, the bank's value function decreases as Θ becomes smaller because the moral hazard constraint is always binding (Eq. (21)). The right panel of Figure 4 shows that the total bank value in the capital ratio policy is higher than under the GKQ policy for any given τ^e. By Eq. (21), we can see that the capital ratio policy has a higher bank value because it raises the total bank assets to a higher level[10].

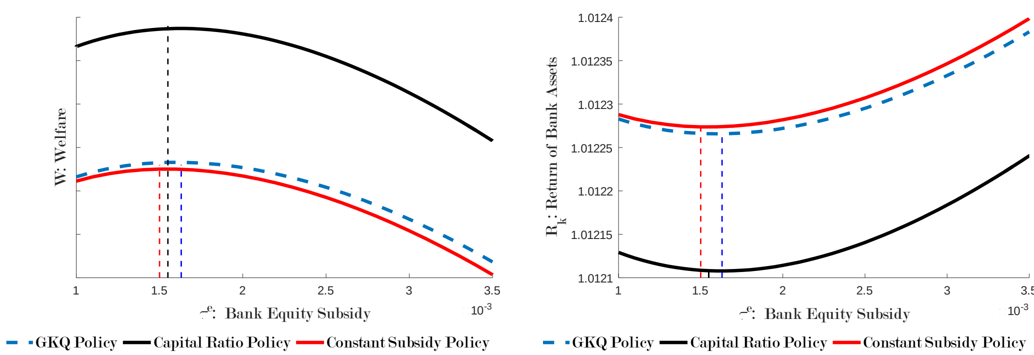
[10]In Eq. (2), the capital stock (K) equals the realized intermediate capital (S) (or bank assets) in steady state because the mean of the financial shock (ψ) is 1.

Figure 4: Bank Value V and Bank Assets QS (Physical Capital QK)



In Figure 5, the welfare ($\mathcal{W}$) first goes up as the central bank increases the subsidy to bank equity (τ^e), but then it goes down when the subsidy exceeds 17 basis points. By closely looking at the return on physical capital (R_k), we find that the highest welfare corresponds to the lowest level of R_k in all policies. This is intuitive: according to the diminishing marginal productivity of capital in the production function, the lowest level of return on physical capital (R_k) corresponds to the highest level of capital stock (see the right-hand panel of Figure 4), which gives the highest output and consumption in steady state.

Figure 5: Welfare $\mathcal{W}$ and Return of Capital R_k



R_k is the return per unit of physical capital and also the risky return on the bank asset. In Figure 5, we can see that R_k first goes down and then goes up as the central bank raises the subsidy to bank equity. To investigate how the return on the bank asset is affected by different channels under different policies, we formulate the following proposition.

Proposition 1 *In the macroprudential policy framework (Eq. (30)) for all three policies, the return on the bank asset can be expressed as a function of the bank capital ratio, the* *subsidy to bank equity and the second moments of variables in steady state.*

$$\begin{aligned} R_k &= \frac{(M_{\mu e} + \tau^e M_v)\left(1 + \varepsilon x + \frac{\kappa}{2}x^2\right)}{M_{\mu s}(\varepsilon + \kappa x)} + \frac{M_v - M_{\mu e}x}{M_{\mu s}} \\ &\equiv S_1 + S_2. \end{aligned} \tag{38}$$

where ε and κ are parameters in the model, $M_{\mu e}$, M_v and $M_{\mu s}$ are second moments of μ_e, v and μ_s respectively, $S_1 = \frac{(M_{\mu e}+\tau^e M_v)\left(1+\varepsilon x+\frac{\kappa}{2}x^2\right)}{M_{\mu s}(\varepsilon+\kappa x)}$ and $S_2 = \frac{M_v - M_{\mu e}x}{M_{\mu s}}$.

Proof. The equations in the banking sector in steady state are

$$v = \Omega(1 + cov(\widehat{\Omega}, \widehat{\Lambda})), \tag{39}$$

$$\mu_s = \Lambda\Omega(R_k - R) + (R_k - R)\Lambda\Omega cov(\widehat{\Lambda}, \widehat{\Omega}) + R_k\Lambda\Omega cov(\widehat{\Lambda}, \widehat{R}_k) + R_k\Lambda\Omega cov(\widehat{\Omega}, \widehat{R}_k), \tag{40}$$

$$\mu_e = \Lambda\Omega(R - R_e) + (R - R_e)\Lambda\Omega cov(\widehat{\Lambda}, \widehat{\Omega}) - R_e\Lambda\Omega cov(\widehat{\Lambda}, \widehat{R}_e) - R_e\Lambda\Omega cov(\widehat{\Omega}, \widehat{R}_e), \tag{41}$$

$$(\mu_s + \mu_e x)(\varepsilon + \kappa x) = \left(1 + \varepsilon x + \frac{\kappa}{2}x^2\right)[\mu_e + \tau^e v], \tag{42}$$

where $cov(\cdot)$ is the covariance between the log-deviated variables, expression (39) is the definition of v_t, (40) is the definition of $\mu_{s,t}$, (41) is the definition of $\mu_{e,t}$, Eq. (42) is the optimal choice of x_t, and all variables are at their stochastic steady states. We can derive (38) from (39), (40), (41) and (42) where

$$M_v = 1 + cov(\widehat{\Omega}, \widehat{\Lambda}),$$

$$M_{\mu e} = -\Lambda\left[(R_e - R) + (R_e - R)cov(\widehat{\Lambda}, \widehat{\Omega}) + R_e cov(\widehat{\Lambda}, \widehat{R}_e) + R_e cov(\widehat{\Omega}, \widehat{R}_e)\right],$$

$$M_{\mu s} = \Lambda\left(1 + cov(\widehat{\Lambda}, \widehat{\Omega}) + cov(\widehat{\Lambda}, \widehat{R}_k) + cov(\widehat{\Omega}, \widehat{R}_k)\right).$$

■

In Eq. (38), these second moments ($M_{\mu e}$, M_v, $M_{\mu s}$) are endogenous and vary as the central bank changes the bank equity subsidy, τ^e.

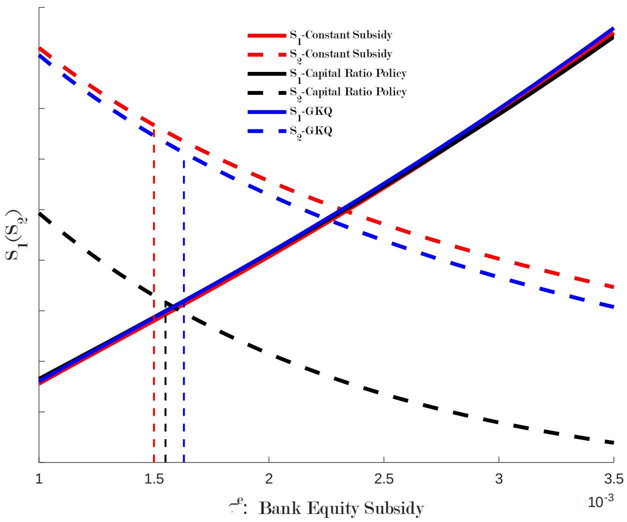
We can see from Eq. (38) that there are two kinds of effects on the return on capital. The first term S_1 is the moral hazard effect on R_k. Since $\theta\left(1 + \varepsilon x + \frac{\kappa}{2}x^2\right)$ is the fraction of bank assets that bank managers divert from the banking sector and $\theta(\varepsilon + \kappa x)$ is the marginal fraction of diversion, and then $\frac{1+\varepsilon x+\frac{\kappa}{2}x^2}{\varepsilon+\kappa x}$ is the inverse of the marginal contribution ratio of x_t on the moral hazard cost. The second term S_2 contains the bank's cost reduction effect when the bank switches from bank deposits to bank equities. To be clear, we rewrite S_2 as

$$S_2 = \frac{M_v - M_{\mu e}x}{M_{\mu s}} = \frac{(M_v - M_{\mu e}x)\,\Omega}{M_{\mu s}\Omega} = \frac{v - \mu_e x}{v + \mu_s}, \tag{43}$$

where the third equality comes from the derivation of Proposition 1. Recalling from (23), (24) and (25), μ_e is the excess cost of the deposit to bank equity and $v + \mu_s$ is the return of the bank asset from an additional net worth tomorrow. As banks raise the proportion of bank equities, the capital ratio goes up. Therefore, $-\mu_e x$ is the banks' cost reduction when they issue more bank equities instead of deposits. When banks' cost of borrowing funds from households decreases, R_k also decreases through S_2.

Figure 6 decomposes the moral hazard effect and the cost reduction effect on R_k in all three policies. When τ^e is small, the return on capital decreases as τ^e increases because the cost reduction effect S_2 is large. However, the moral hazard effect $\frac{1+\varepsilon x+\frac{\kappa}{2}x^2}{\varepsilon+\kappa x}$ is increasing when the bank capital ratio goes up. As a result, R_k is a U-shaped curve as the central bank raises τ^e. Figure 6 shows that the moral hazard effect is nearly the same in all three policies, but the cost reduction effect reduces R_k more in the capital ratio policy.

Figure 6: Moral Hazard Effect v.s. Cost Reduction Effect



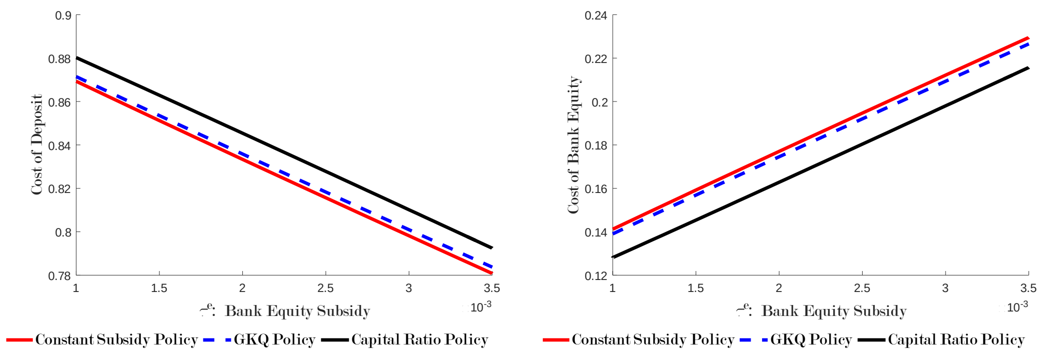
The cost reduction effect in Eq. (43) can be rewritten as a ratio of the bank's cost to the bank's profit:

$$S_2 = \frac{v - \mu_e x}{v + \mu_s} = \frac{(1 - x)\, v}{v + \mu_s} + \frac{x\,(v - \mu_e)}{v + \mu_s}.$$

In the numerator, there is the bank's cost of issuing deposits ($(1-x)\,v$) and bank equities ($x\,(v-\mu_e)$) with the bank capital ratio (x). The denominator $v+\mu_s$ is the return of the bank asset from an additional future net worth. If this ratio is low, the bank gains more profit from the additional net worth tomorrow. Now we see the reason why the GKQ policy is not optimal: the central bank targets v which is the cost of banks issuing deposits, but it does not consider the policy effect on the profit of bank assets.

Figure 7 illustrates how different policies affect the cost of banks issuing deposits and bank equities. While the GKQ policy and the constant subsidy policy perform better in decreasing the cost of the bank deposit ($\frac{(1-x)v}{v+\mu_s}$), the capital ratio policy reduces the cost of bank equities ($\frac{x(v-\mu_e)}{v+\mu_s}$) more. The cost reduction of the bank equity dominates the cost increase of the bank deposit. As a result, the capital ratio policy performs better for the cost reduction effect than the GKQ policy. This effect lowers the return of capital because the bank's cost for issuing funds from households with a balance sheet structure x is decreased more in the capital ratio policy.

Figure 7: Bank's Cost for Issuing Deposits and Bank Equities

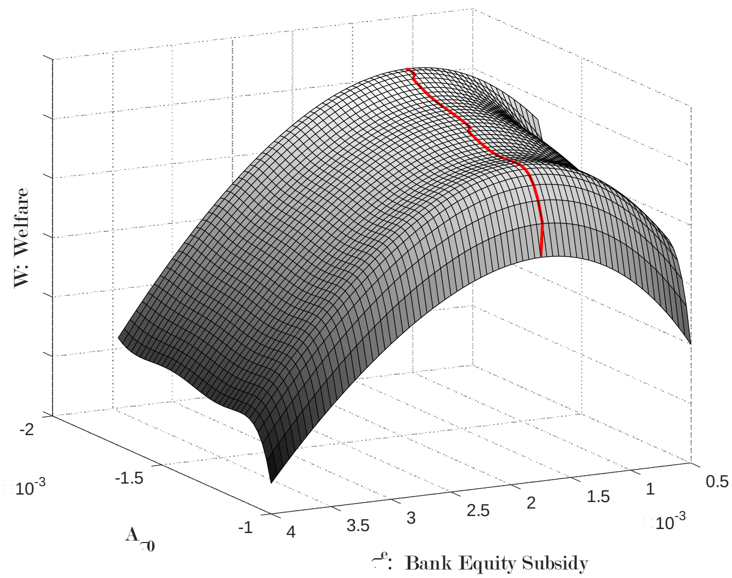


4.2 Optimal Parameters in New Policy

This section provides the optimal policy parameters $(\tau_0, A_{\tau 0})$ that give the highest welfare in the capital ratio policy. By a grid search of two parameters, the optimal point is found at $(\tau_0^*, A_{\tau 0}^*) = (0.0209, -0.0017)$.

Figure 8 shows the welfare surface with different policy parameters in the capital ratio policy rule. The red line on the surface shows the optimal welfare point for each value of $A_{\tau 0}$. The Y-axis is the bank equity subsidy τ^e instead of parameter τ_0 because it is easier to compare the policy effect on welfare when the balance sheet channel is fixed. Figure 8 shows that the smaller value of $A_{\tau 0}$ gives a higher welfare.

Figure 8: Welfare[11]



Note: The optimal point is when $\tau_0 = 0.0209$, and it corresponds to a $\tau^e = 0.0016$.

Although the optimal parameter $A^*_{\tau 0}$ is negative, the bank equity subsidy τ^e_t is positive in steady state. The optimal policy parameters suggest that the central bank offers a progressive subsidy to the per unit of bank equity with respect to the aggregate bank capital ratio. When we examine how the central bank operates the policy after the shock in Section 3, we find that the capital ratio policy works as a countercyclical capital ratio requirement. That is, the central bank requires a higher capital ratio at normal times but lowers the requirement once a financial crisis occurs.

As in Section 4.1, the following analysis evaluates variables in steady state from the general equilibrium perspective. To shut down the balance sheet channel, τ^e is fixed across different policy parameters. We follow the steps in Section 4.1 and investigate the moral hazard effect and the cost reduction effect on the return of capital (R_k) with different values of $A_{\tau 0}$. Here we compare three different parameter scenarios in the capital ratio policy when $A_{\tau 0} = -0.0017$, -0.0014 and -0.001.

[11]The optimal policy parameter is at the boundary of the Blanchard Kahn conditions. However, when the policy maker chooses the parameter values within the interval $A_{\tau 0} \in (-0.0017, 0)$ and $\tau_0 \in (-0.0209, -0.0045)$, the capital ratio policy performs better than the GKQ policy. So the policy maker can choos the parameter that is away from the boundary but still outperforms the GKQ policy. The author acknowledge the boundary issue in the capital ratio policy, but this is not the main focus of the paper.

Figure 9: Capital Ratio x and Moral Hazard Cost Θ

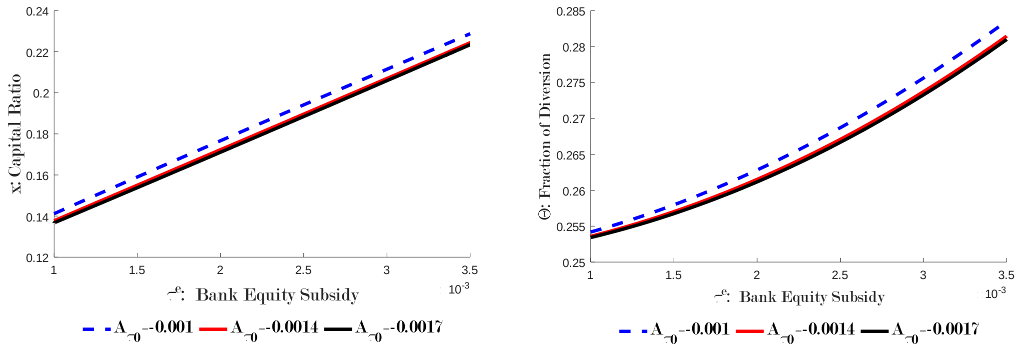


Figure 9 shows that the smaller parameter value of $A_{\tau 0}$ leads banks to have a lower capital ratio and a lower fraction of diversion. A lower value of $A_{\tau 0}$ creates a smaller moral hazard cost because the bank's extra cost of issuing bank deposits (μ_e) is reduced (see the top-right panel of Figure 10). Therefore, banks have less incentives to switch from deposits to bank equities. Moreover, a smaller value of parameter $A_{\tau 0}$ in the capital ratio policy raises the net profit of the bank asset, $\mu_s + x\mu_e$, which makes each unit of net worth more valuable in the future (see the bottom-right panel of Figure 10).

Figure 10: Variables in the Banking Sector

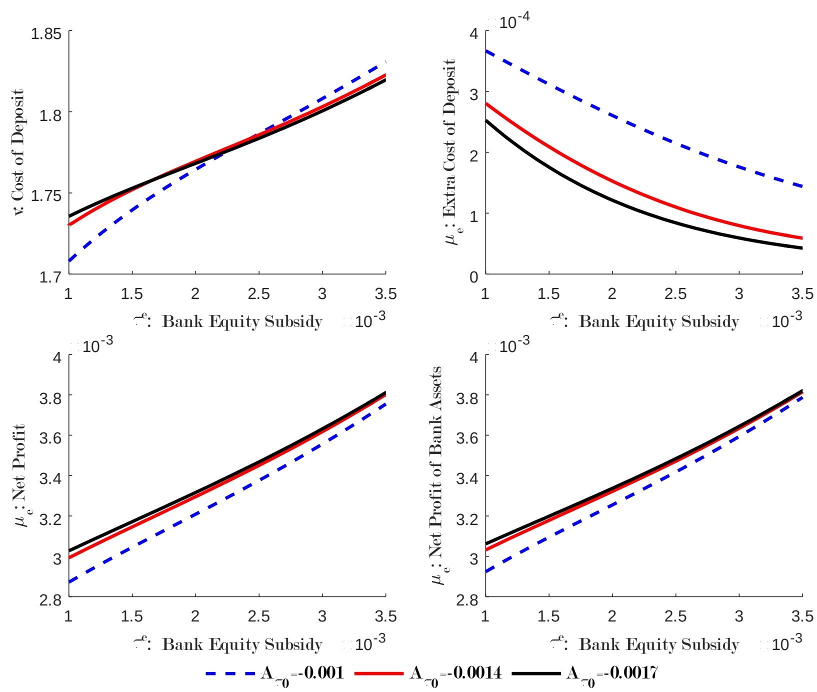


Figure 11: Bank Value V and Bank Assets QS (Physical Capital QK)

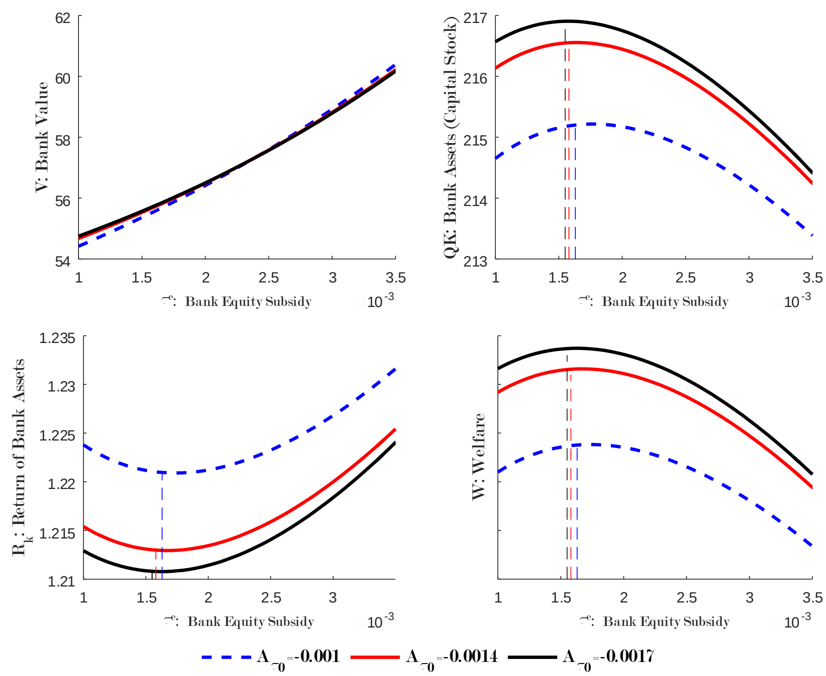
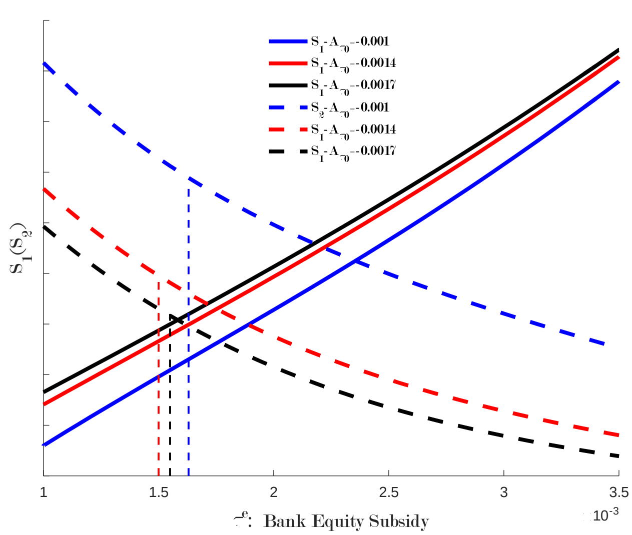


Figure 11 shows that the capital ratio policy with $A_{\tau 0} = -0.0017$ gives the best performance of reducing the return on capital (R_k) given any level of bank equity subsidy (τ^e), thus raising the capital stock (K) in steady state and improving welfare. From the previous analysis in Section 4.1, welfare is highest when R_k is at the lowest level in the capital ratio policy. To figure out how different parameters affect welfare, the policy effect is decomposed into the moral hazard effect and the cost reduction effect in Figure 12.

Figure 12: Moral Hazard Effect S_1 v.s. Cost Reduction Effect S_2



As compared to other parameter scenarios, the policy with a lower value of $A_{\tau 0}$ performs worse on the moral hazard effect, but better on the cost reduction effect. When $A_{\tau 0}$ is more negative, the cost reduction effect dominates the moral hazard effect. As a result, when $A_{\tau 0}$ has the smallest value, the return per unit of capital is at the lowest level and therefore the economy achieves the highest welfare.

The optimal parameters in the capital ratio policy are related to the macroprudential policy literature. When the central bank gives a progressive subsidy to the bank equity with respect to the bank capital ratio, it guarantees a higher capital ratio before the financial shock but a lower level after the crisis. The policy implication is consistent with the time-varying capital ratio requirements in Kashyap and Stein (2004). At normal times, the central bank makes the capital ratio relatively high using a higher level of bank equity subsidy. When the financial crisis occurs, the asset price deterioration makes the bank capital ratio much lower. In this situation, the central bank offers less subsidy to bank equities. Equivalently, it lowers the capital ratio requirement for individual banks after the financial crisis. Moreover, the new macroprudential policy works similarly as the macroprudential capital policy from Bank of England, such as Harimohan and Nelson (2012).

5 Conclusion

This paper provides an alternative macroprudential policy in the framework of GKQ. The new policy chooses the subsidy of bank equity proportional to the bank capital ratio. The new policy has two advantages as compared to the GKQ policy. First, banks' balance sheet structure is observable for the central bank while the shadow cost of the deposit cannot be easily observed. Therefore, the new policy is easier to implement in practise than the GKQ policy. Second, the new policy is welfare dominant to the GKQ policy.

We investigate how the GKQ policy and the new policy improve welfare through different channels. We find that both policies improve welfare because they incentivize banks to issue a higher fraction of bank outside equities. From the perspective of the bank, it is cheaper to issue bank equities than deposits, so the policy reduces the bank's cost of borrowing funds from households. In comparison, we keep the bank balance sheet channel across different policies and compare the moral hazard channel and the cost reduction channel. The results show that the moral hazard effect on welfare is very similar between the new policy and the GKQ policy. On the other hand, the new policy performs better at reducing the bank's cost for getting funds from households. It reduces the returns on capital, raises the capital stock, output and consumption, and hence improves welfare. This policy implementation is consistent with the countercyclical capital ratio requirement in the macroprudential literature.

6 Appendix

6.1 System of Equations with the New Policy

$$Y_t = (\psi_t S_{t-1})^{\alpha} L_t^{1-\alpha}. \tag{44}$$

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t. \tag{45}$$

$$S_t = \psi_t \left[(1-\delta)S_{t-1} + I_{t-1}\right]. \tag{46}$$

$$R_{t+1}\mathbb{E}_t\left(\Lambda_{t,t+1}\right) = 1. \tag{47}$$

$$\mathbb{E}_t\left(\Lambda_{t,t+1}R_{e,t+1}\right) = 1. \tag{48}$$

$$j_t = \frac{J_t}{J_{t-1}}. \tag{49}$$

$$J_t = C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}. \tag{50}$$

$$(1-\alpha)\left[1 - \beta h \mathbb{E}_t\left(j_{t+1}^{-\gamma}\right)\right] Y_t = \chi L_t^{1+\varphi}. \tag{51}$$

$$\mathbb{E}_t\Lambda_{t,t+1} = \beta\frac{\mathbb{E}_t\left(j_{t+1}^{-\gamma}\right) - \beta h \mathbb{E}_t\left(j_{t+1}^{-\gamma}j_{t+2}^{-\gamma}\right)}{1 - \beta h \mathbb{E}_t\left(j_{t+1}^{-\gamma}\right)}. \tag{52}$$

$$R_{e,t} = \frac{\left[\alpha\left(\frac{L_t}{\psi_t S_{t-1}}\right)^{1-\alpha} + (1-\delta)q_t\right]\psi_t}{q_{t-1}}. \tag{53}$$

$$R_{k,t} = \frac{\left[\alpha\left(\frac{L_t}{\psi_t S_{t-1}}\right)^{1-\alpha} + (1-\delta)Q_t\right]\psi_t}{Q_{t-1}}. \tag{54}$$

$$Q_t = 1 + f(\frac{I_t}{I_{t-1}}) + \frac{I_t}{I_{t-1}}f'(\frac{I_t}{I_{t-1}}) - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'(\frac{I_{t+1}}{I_t})\right]. \tag{55}$$

$$N_t = \sigma\left[\left(R_{k,t} - x_{t-1}R_{e,t} - R_t + x_{t-1}R_t\right)Q_{t-1}S_{t-1} + R_t N_{t-1}\right] + (1-\sigma)\xi R_{k,t}Q_{t-1}S_{t-1}. \tag{56}$$

$$\Theta_t = \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right). \tag{57}$$

$$N_t\phi_t = Q_t K_t. \tag{58}$$

$$\phi_t = \frac{v_t}{\Theta_t - (\mu_{st} + x_t\mu_{et})}. \tag{59}$$

$$v_t = \mathbb{E}_t(\Lambda_{t,t+1}\Omega_{t+1})R_{t+1}. \tag{60}$$

$$\mu_{s,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\right]. \tag{61}$$

$$\mu_{e,t} = \mathbb{E}_t \left[\Lambda_{t+1} \Omega_{t+1} \left(R_{t+1} - R_{e,t+1} \right) \right]. \tag{62}$$

$$\Omega_t = (1-\sigma) + \sigma \left[(\mu_{s,t} + x_t \mu_{e,t}) \phi_t + \upsilon_t \right]. \tag{63}$$

$$\theta \left(\mu_{s,t} + \mu_{e,t} x_t \right) (\varepsilon + \kappa x_t) = \theta \left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2 \right) \left[\mu_{e,t} + \upsilon_t \left(\tau_0 x_t + A_{\tau 0} \right) \right] \tag{64}$$

$$\left(1 + \tau_t^k \right) Q_t S_t = N_t + (1 + \tau_t^e) \, q_t e_t + D_t \tag{65}$$

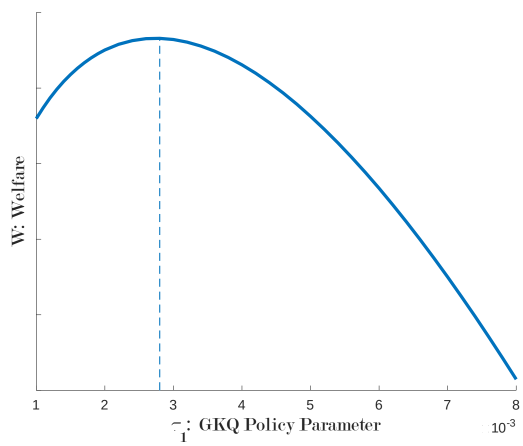
$$x_t = \frac{q_t e_t}{Q_t S_t} \tag{66}$$

$$\tau_t^e = \tau_0 x_t + A_{\tau 0} \tag{67}$$

$$\tau_t^k = x_t \tau_t^e \tag{68}$$

6.2 Optimal Parameter in the GKQ Policy

Figure 13: Welfare of the GKQ Policy



6.3 Optimization Problem of Households

The representative household's utility function is

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} \left(C_\tau - h C_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi} \right)^{1-\gamma}, \tag{69}$$

subject to the budget constraint

$$\Xi_t = C_t + D_{h,t} + q_t e_t - W_t L_t - \Pi_t - R_t D_{h,t-1} - \left[Z_t + (1-\delta) q_t \right] \psi_t e_{t-1} \leq 0. \tag{70}$$

The Lagrangian is

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \frac{1}{1-\gamma} \left(C_\tau - h C_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi} \right)^{1-\gamma} - \lambda_\tau \Xi_\tau \right\}. \tag{71}$$

The representative household chooses variables $(C_t, L_t, D_{h,t}, e_t)$ to maximize (69). The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial C_t} = \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right)^{-\gamma} - \lambda_t - \beta h \mathbb{E}_t \left(C_{t+1} - hC_t - \frac{\chi}{1+\varphi}L_{t+1}^{1+\varphi}\right)^{-\gamma} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = -\chi \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right)^{-\gamma} L_t^{\varphi} + \lambda_t W_t = 0,$$

$$\frac{\partial \mathcal{L}}{\partial D_{h,t}} = \mathbb{E}_t \left(-\lambda_t + \beta R_{t+1}\lambda_{t+1}\right) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = -\lambda_t q_t + \beta \mathbb{E}_t \left[\lambda_{t+1}\left(Z_{t+1} + (1-\delta) q_{t+1}\psi_{t+1}\right)\right] = 0.$$

From the above equations, we can derive Eqs. (8), (9), (10) and (13) in Section 2.1.

6.4 Optimization Problem of the Banking Sector

We use the techniques of Bellman equations to solve the commercial bank's optimization problem because the value function appears in the moral hazard constraint. Here we show two different ways to derive the specific form of the value function.

6.4.1 Guess and Verify Method in Gertler et al. (2012)

We simplify the net worth accumulation function by Eqs. (15) and (18)

$$n_t = \left[R_{k,t} - R_t(1-x_t) - R_{e,t}x_{t-1}\right] Q_{t-1}s_{t-1} + R_t n_{t-1}.$$

Here x_t, s_t are the control variables and n_t is the state variable. The Bellman equation is

$$V_{t-1}(n_{t-1}) = \mathbb{E}_{t-1}\Lambda_{t-1,t}\left\{(1-\sigma)n_t + \sigma \max_{s_t, x_t}\left[V_t(n_t)\right]\right\}. \tag{72}$$

The guess solution is

$$V_t(n_t) = (\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t. \tag{73}$$

We need to verify that the solution satisfies the expression (72) for all t.

The bank maximizes the objective function subject to the moral hazard constraint, so the Lagrangian is constructed as

$$\mathcal{L} = (1+\lambda_t)\left[(\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t\right] - \lambda_t\theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right) Q_t s_t.$$

The first order conditions with respect to x_t and s_t are

$$(1+\lambda_t) = \frac{\lambda_t\theta\left(\varepsilon + \kappa x_t\right)}{\mu_{e,t}}. \tag{74}$$

$$(1+\lambda_t)(\mu_{s,t} + x_t\mu_{e,t}) = \lambda_t\theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right). \tag{75}$$

The moral hazard constraint is always binding, so we have

$$V_t(n_t) = (\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t = \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right) Q_t s_t.$$

Substituting (74) into (75), we derive the implicit function of $\left(x_t, \mu_{s,t}, \mu_{e,t}\right)$,

$$(\varepsilon + \kappa x_t)\,(\frac{\mu_{s,t}}{\mu_{e,t}} + x_t) = 1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2.$$

Substitute (73) into the value function $V_t(n_t)$,

$$(\mu_{s,t} + x_t \mu_{e,t}) Q_t s_t + v_t n_t = \mathbb{E}_t \Lambda_{t,t+1} \left\{(1-\sigma) + \sigma\left[(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1})\phi_{t+1} + v_{t+1}\right]\right\} n_{t+1}.$$

Denoting $\Omega_{t+1} = (1-\sigma) + \sigma\left[(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1})\phi_{t+1} + v_{t+1}\right]$, the value function becomes

$$(\mu_{s,t} + x_t \mu_{e,t}) Q_t s_t + v_t n_t = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \left(\left[R_{k,t+1} - R_{t+1}(1 - x_{t+1}) - R_{e,t+1} x_t\right] Q_t s_t + R_{t+1} n_t\right). \quad (76)$$

So if we denote variables $(v_t, \mu_{s,t}, \mu_{e,t}, \Omega_{t+1})$ as Eqs. (23), (25), (26) and (24) in Section 2.4, both sides of Eq. (76) are identical for all t.

$$\begin{aligned} LHS &= \mathbb{E}_t \left\{\Lambda_{t,t+1}\Omega_{t+1}\left[\left(R_{k,t+1} - R_{t+1}\right) Q_t s_t + x_t\left(R_{t+1} - R_{e,t+1}\right) Q_t s_t + R_{t+1} n_t\right]\right\} \\ &= \mathbb{E}_t \Lambda_{t,t+1}\Omega_{t+1}\left(\left[R_{k,t+1} - \left(1 - x_t\right) R_{t+1} - x_t R_{e,t+1}\right] Q_t s_t + R_{t+1} n_t\right) \\ &= RHS. \end{aligned}$$

So the Bellman equation can be satisfied for any t with the specific form (73), and this method requires the correct hunch of the functional form.

6.4.2 Derivation Method without Guessing the Functional Form

The bank's objective function is

$$V_t(n_t) = \mathbb{E}_t \left[\sum_{\tau=t+1}^{\infty} (1-\sigma)\sigma^{\tau-t-1} \Lambda_{t,\tau} n_\tau\right].$$

The Lagrangian with the net worth accumulation function is

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t+1}^{\infty} \sigma^{\tau-t-1} \Lambda_{t,\tau} \left\{(1-\sigma) n_\tau + \Omega_\tau g(n_\tau, n_{\tau-1}, x_{\tau-1}, s_{\tau-1})\right\}.$$

where $g(n_t, n_{t-1}, x_{t-1}, s_{t-1})$ is the net worth accumulation function

$$g(n_t, n_{t-1}, x_{t-1}, s_{t-1}) = \left[R_{k,t} - R_t(1 - x_{t-1}) - R_{e,t} x_{t-1}\right] Q_{t-1} s_{t-1} + R_t n_{t-1} - n_t = 0.$$

The first order condition with respect to n_{t+1} is

$$\begin{aligned} \Omega_{t+1} &= (1-\sigma) + \sigma \frac{\Lambda_{t,t+2}}{\Lambda_{t,t+1}\Lambda_{t+1,t+2}} \Lambda_{t+1,t+2} \Omega_{t+2} R_{t+2} \\ &\quad + \sigma \phi_t \frac{\Lambda_{t,t+2}}{\Lambda_{t,t+1}\Lambda_{t+1,t+2}} \left[\Lambda_{t+1,t+2}\Omega_{t+2}\left(R_{k,t+2} - R_{t+2}\right) + x_{t+1}\Lambda_{t+1,t+2}\Omega_{t+2}\left(R_{t+2} - R_{e,t+2}\right)\right]. \end{aligned}$$

If we use the same notations as Eqs. (23), (24), and (25), we have

$$\Omega_{t+1} = (1-\sigma) + \sigma\left[\phi_t\left(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1}\right) + v_{t+1}\right].$$

Recall the Bellman equation,

$$V_t(n_t) = \mathbb{E}_t (1-\sigma)\Lambda_{t,t+1} n_{t+1} + \sigma \mathbb{E}_t \Lambda_{t,t+1} V_{t+1}(n_{t+1}). \quad (77)$$

Rearrange the net worth accumulation function as

$$n_{t+1} - \Gamma_t n_t = 0,$$

where

$$\Gamma_t = [R_{k,t+1} - R_{t+1}(1 - x_t) - R_{e,t+1}x_t]\,\phi_{t+1} + R_{t+1}.$$

Letting Ω_{t+1} denote the price of net worth at time $t+1$, the bank's value function V_t equals the discounted value of net worth tomorrow,

$$V_t(n_t) = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}n_{t+1}. \tag{78}$$

We substitute it into the Bellman equation (77) to get the functional form of Ω_{t+1}, and we will verify later that Ω_{t+1} is the price of net worth tomorrow.

$$\begin{aligned}\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}n_{t+1} &= \mathbb{E}_t\left((1-\sigma)\Lambda_{t,t+1}n_{t+1} + \sigma\mathbb{E}_t\Lambda_{t,t+1}\left[E_{t+1}\Lambda_{t+1,t+2}\Omega_{t+2}n_{t+2}\right]\right)\\ &= \mathbb{E}_t\Lambda_{t,t+1}n_{t+1}\left[(1-\sigma) + \sigma\left(\Lambda_{t+1,t+2}\Omega_{t+2}\Gamma_{t+1}\right)\right].\end{aligned}$$

Since the equality always holds for any t, we can derive the exact form of Ω_{t+1} by comparing both sides of the equation above after rearrangements,

$$\Omega_{t+1} = (1-\sigma) + \sigma\Lambda_{t+1,t+2}\Omega_{t+2}\Gamma_{t+1}. \tag{79}$$

Now we show that Ω_{t+1} is the price of net worth at time $t+1$. The Lagrangian is

$$\mathcal{L} = \mathbb{E}_t\left[\sum_{\tau=t+1}^{\infty}\sigma^{\tau-t-1}\Lambda_{t,\tau}\left[(1-\sigma)n_\tau - \Omega_\tau\left[n_\tau - \Gamma_{\tau-1}n_{\tau-1}\right]\right]\right].$$

We differentiate $\mathcal{L}$ with respect to n_{t+1},

$$\mathbb{E}_t\Lambda_{t,t+1}(1-\sigma) - \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1} + \sigma\mathbb{E}_t\Lambda_{t,t+2}\Omega_{t+2}\Gamma_{t+1} = 0,$$

which is exactly the same as how Ω_{t+1} is defined in the previous method.

To get the closed form of the value function V_t, we substitute (79) into (78)

$$\begin{aligned}V_t(n_t) &= \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Gamma_t n_t\\ &= \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}n_t + \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\phi_t n_t + \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}x_t\left(R_{t+1} - R_{e,t+1}\right)\phi_t n_t.\end{aligned}$$

Compared with the value function (22) in the previous method, they are the same.

6.5 Stochastic Steady State

The steps of solving the stochastic steady state are shown as follows. First, we linearize the model around the deterministic steady state. We use the steady state and the non-linear system of equations to derive the second moments of the deterministic steady state. Second, we substitute the second moments back into the non-linear system to derive a new steady state. Third, we can derive new second moments based on the new steady state. We keep iterating and updating the steady state values until the steady state generates the second moments which lead the non-linear system back to the same steady state.

The model can be described as n nonlinear equations with n endogenous variables, where $\mathbf{y}^+_{t+1}$ is a vector of forward-looking variables, $\mathbf{y}_t$ is static variables and $\mathbf{y}^-_{t-1}$ is predetermined variables.

$$\mathbb{E}_t\left[\mathbf{f}(\mathbf{y}^+_{t+1}, \mathbf{y}_t, \mathbf{y}^-_{t-1})\right] = \mathbf{0}.$$

If we have the ith equation with forward-looking variables u_{t+1}, we the do second order approximation around the expected value,

$$\Phi_i(\mathbb{E}_t u_{t+1}) = \mathbf{f}_i\left(\mathbb{E}_t u_{t+1}\right) + \mathbf{f}'_i\mathbb{E}_t(u_{t+1} - \mathbb{E}_t u_{t+1}) + \frac{1}{2!}\mathbb{E}_t\mathbf{f}''_i\mathbb{E}_t(u_{t+1} - \mathbb{E}_t u_{t+1})^2 + o(\cdot) \simeq 0,$$

where $\mathbf{f}'_i(\cdot)$ and $\mathbf{f}''_i(\cdot)$ are the first and the second derivative at the point $\mathbb{E}_t u_{t+1}$. $\mathbb{E}_t(u_{t+1} - E_t u_{t+1})^2$ is the second moment of the dynamic system around the stochastic steady state.

6.6 Welfare and Consumption Equivalence

Welfare $\mathcal{W}_t$ in the model is the unconditional steady state of the representative household's lifetime utility

$$\mathcal{W}_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau, L_\tau\right). \tag{80}$$

We take second order approximation of the utility function around the stochastic steady state, so the volatilities of consumption and labour are included. If we write it in recursive form,

$$\mathcal{W}_t = U\left(C_t, L_t\right) + \beta \mathbb{E}_t \mathcal{W}_{t+1}.$$

After simplification, we have

$$\mathcal{W} = \frac{U(C, L) + \beta COV(C, L)}{1-\beta},$$

where $COV(C, L)$ is the second moments of C and L. To be specific,

$$\begin{aligned} COV(C, L) \quad = \quad & \frac{1}{2}\left[\frac{\partial^2 U}{\partial C_{t+1}^2}\right]_{C,Y} var(C_{t+1}) + \frac{1}{2}\left[\frac{\partial^2 U}{\partial L_{t+1}^2}\right]_{C,Y} var(L_{t+1}) \\ & + \left[\frac{\partial^2 U}{\partial C_{t+1}\partial L_{t+1}}\right]_{C,Y} cov(L_{t+1}, C_{t+1}). \end{aligned}$$

$\left[\frac{\partial^2 U}{\partial C_{t+1}^2}\right]_{C,Y}$ denotes the second derivative with respect to C and Y at steady state.

We use consumption equivalents to compare welfare between different policy scenarios. Denote Γ as the percentage increase of consumption needed in the baseline model to reach the same level of welfare in macroprudential policies. Denote $\mathcal{W}^B$ as the welfare value in the baseline model, $\mathcal{W}^{GKQ}$ as the welfare value in the GKQ policy and Γ^{GKQ} as the additional percentage of consumption, we have

$$\mathcal{W}^{GKQ} = \frac{U\left((1+\Gamma^{GKQ})C^B, L^B\right) + \beta COV(C^B, L^B)}{1-\beta}, \tag{81}$$

where C^B and L^B is the stochastic steady state values of consumption and labour in the baseline model.

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